EFFECT OF THE ADIABATIC EXPONENT ON THE REFLECTION OF SHOCK WAVES

G. M. Arutyunyan, V. A. Belokon',
and L. V. Karchevskii

The effect of the adiabatic exponent on certain features of shock-wave reflection is analyzed with particular reference to the nature of the dependence of the critical angle on the incident wave intensity. The latter is shown to increase with increasing shock-wave intensity. Limit cases of weak shock waves at any arbitrary adiabatic exponent and of strong shock waves with the adiabatic exponent equal to unity are analytically investigated. Results of calculations of the critical angle for various adiabatic exponents throughout the possible range of incident wave intensities are presented.

1. The reflection of a shock wave from another or from a contact discontinuity (including a wall) depends on the adiabatic exponent, particularly when the wave is sufficiently strong.

To simplify theoretical consideration of certain aspects of this problem the medium is assumed to be perfect, so that its equation of state together with the Rankin-Hugoniot adiabatic equation can be written as

$$\gamma = \frac{c_p}{c_v} = 1 + \frac{pV}{E} = 1 + \frac{2}{f} = \frac{(p_1 - p_0)(V_0 + V_1)}{(p_1 + p_0)(V_0 - V_1)}. \tag{1.1}$$

Here the notation is either obvious, or will be explained in the following: subscripts 0 and 1 relate to the state of the medium in front of and behind the shock, respectively, and it is assumed that $\gamma_1 = \gamma_0$ or $f_1 = f_0$.

The structure of the shock, whose width is assumed to be zero, is disregarded, i.e., dissipative effects, such as viscosity, thermal conductivity, etc., are neglected. In this formulation the assumption of absence of heat exchange with the wall becomes superfluous.

In spite of this idealization of the medium it is reasonable to expect the results of this analysis to be valid not only for media having various γ = const but, also, for those with varying adiabatic exponents. This, for example, is to be expected under conditions in which an approximating substitution of a certain mean value for a variable γ is acceptable. However, since compression of a medium by a strong shock wave depends only on its specific heat behind the shock, the problem is virtually reduced to the case of γ = const.

2. The simplest explanation of the existence of adiabatic exponents of various magnitudes is provided by the assumption that $2/(\gamma - 1) = f$ is the number of degrees of freedom of the medium molecules in the so-called equidistribution approximation [1], according to which part of the possible degrees of freedom is unconstrained and the rest totally constrained, frozen. In this approximation f may be, generally speaking, any of the series of natural numbers. Properties of a real gas are, however, better approximated by the assumption of a continuous set of possible values of f or γ, while formally maintaining the "equidistribution" concept.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 1, pp. 62-66, January-February, 1970. Original article submitted January 30, 1969.

© 1972 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for $15.00.

In other words, we assume that $1 \ll f \ll \infty$.

The traditional cases are $f=3$ (all degrees of translational freedom unconstrained) and $f=5$ (plus two degrees of rotational freedom).

With increasing temperatures and decreasing densities unconstrained degrees of freedom of oscillation become important, i.e., with $f \to 7$ in the case of diatomic molecules (since the energy of oscillations consists of kinetic and potential components). The omitted value of $f=6$ relates to any substance radiating at sufficiently high temperature and, more precisely, to an ultrarelativistic gas.

In air and other gases within certain ranges of shock velocities dissociation and ionization lead to maximum values of f, i.e., minimum γ. For example, the shock wave velocity D = 16 km/sec in an atmosphere of a density 10^{-6} of the standard density of air at 210 °K (an altitude of 97 km) corresponds to $f = 21.3$ or $\gamma = 1.09$. Hence the interest in the asymptotic, "Newtonian," value of $\gamma = 1$ when $f \to \infty$.

Contrary to Sommerfeld's assertion [2] the values $f = 2$ and $f = 1$ can also be substantiated on physical grounds. The case of $f = 2$ or $\gamma = 2$ could be possibly interpreted as one of total suppression of one degree of translational freedom of every charged particle in an ionized gas slowing across frozen magnetic field lines. The second degenerated case of $f = 1$ or $\gamma = 3$ corresponds to the suppression of two degrees of translational freedom of gas molecules flowing in the direction of magnetic field lines.

We would stress that this interpretation of $f = 1$ ($\gamma = 3$) and $f = \gamma = 2$ has a definite meaning in the case of one-dimensional flows only (plane, cylindrical and spherical shock waves).

It is desirable to investigate, in addition to the [behavior of] ionized gas in a magnetic field, the validity of $f = 1$ for nonunivariate reflection of a shock wave in the "Landau-Stanyukovich gas" [3] in the scattered products of chemical explosives, whose univariate motion is satisfactorily idealized by assuming $\gamma = 3$).

3. Before considering the effect of the adiabatic exponent on the "oblique" reflection of shock waves, let us take note of the properties of "normal" reflection according to the law (first established by Hugoniot in 1885)

$$\frac{p_2}{p_1} = \frac{f+3-(p_0/p_1)}{1+(f+1)\,p_0/p_1} = \begin{cases} 1 + p_0^{-1}\Delta p & (\Delta p \ll p_0) \\ f+3 & (\Delta p \gg p_0) \end{cases} \tag{3.1}$$

which clearly shows the influence of the effective number of degrees of freedom on the reflection of strong shock – an effect which is absent in the reflection of weak shocks. While individual properties of gas do not (in this approximation) affect the reflection of weak shocks, the detection of reflection of a strong shock makes possible to determine without difficulty the equation of state of a medium. In the trivial case of a perfect gas it is sufficient to determine (in the system of coordinates of the unperturbed medium) the velocities D_0 and D_1 of the incident and the reflected waves, respectively. The Mach number of a reflected strong incident wave is $(f + 2)^{1/2}$; hence

$$f = \frac{2D_0}{D_1} - 1$$

or

$$1 \leqslant \frac{D_0}{D_1} = \frac{f+1}{2} < \infty \quad \text{for. } 1 \leqslant f < \infty\,. \tag{3.2}$$

The law of temperature variation behind a reflected shock

$$2 \leqslant \frac{T_2}{T_1} = 2\left(1 + \frac{1}{f+2}\right) \leqslant \frac{8}{3} \tag{3.3}$$

is less sensitive to the adiabatic exponent, i.e., to f.

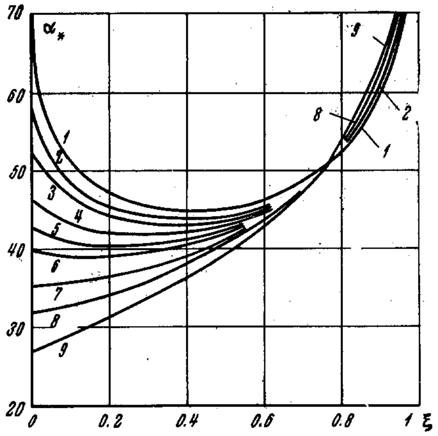

Fig. 1

Measurements of velocities of the incident and reflected shocks provide the necessary minimum of information for the formulation of the equation of state of a medium. The most complete information on the thermal state of a medium in motion would be provided by methods of quantum optics applied to the direct determination of the velocity distribution of gas molecules in the hydrodynamic velocity field.

4. Systematic theoretical and experimental investigations of the dependence of the critical incidence angle of a shock wave (beyond α_*, a regular reflection is impossible) on the wave intensity was, until recently, confined to $\gamma = 1.4$, although later publications [4, 5] provide certain data on the [functional] dependence $\alpha_*(\gamma)$ in strong shock waves, as well as general considerations on the effect of the adiabatic exponent on the pattern of Mach reflection. (See V. A. Belokon', "Problems of phenomenological and statistical thermodynamics of shock waves," dissertation OPM-IPM, 1964-1967.) The use of experimental values of α_* for the determination of the equation of state of a medium (trivial in the case of f = const) was suggested in [6]. One of the subtleties here is the fact that, while a regular reflection is not possible for $\alpha > \alpha_*$, a Mach reflection is nevertheless possible within a certain range of $\alpha < \alpha_*$ [7].

Inadequate investigation of the Mach reflection and the lack of even superficial analysis of the many effects predicted by known formulas may be, to a certain extent, due to the comparative newness of this subject, since such reflections came to be understood only after the publication of works [8, 9]. Incidentally, the formula defining the dependence of the critical angle on wave intensity, given in the classic monographs [7, 10], are incorrect, as noted by K. E. Gubkin.

An analysis of the [functional] dependence $\alpha_*(p_0/p_1, \gamma)$ for the range of $1 \leq \gamma \leq 3$ is given below. The generality of results is limited by the neglect of the (quantitative) dependence of γ on $p_0/p_1 = \xi$.

When analyzing the dependence of α_* on the intensity of an incident shock wave, it is convenient to represent the intensity as $1/\xi = p_1/p_0$. An analysis of the [functional] dependence $\alpha_*(\gamma)$ for asymptotically strong shocks ($\xi = 0$) for certain values of γ is given, e.g., in [4].

The analysis of this dependence for an arbitrary intensity of the incident shock had necessitated a much more thorough examination of calculation results.

Equation

$$ax^3 + bx^2 + cx + d = 0, \qquad x = \mathrm{tg}^2\,\alpha_*, \tag{4.1}$$

which can be derived from the theory of regular reflections, was taken as the basis of calculations. Here

$$a = \gamma + 1$$

$$b = \frac{2\gamma[(\gamma-1)\xi + (\gamma+1)]}{(\gamma+1)\xi + (\gamma-1)} - \frac{[(\gamma-1) + (\gamma+1)\xi]^2}{8\gamma(1-\xi)}$$

$$c = \frac{(\gamma-1)[(\gamma+1) + (\gamma-1)\xi]^2}{[(\gamma-1) + (\gamma+1)\xi]^2} - \frac{[(\gamma+1) + (\gamma-1)\xi]^2}{4\gamma(1-\xi)}$$

$$d = -\frac{[(\gamma+1) + (\gamma-1)\xi]^4}{8\gamma(1-\xi)[(\gamma-1) + (\gamma+1)\xi]^2}$$

The substitution $y = x + b/3a$ transforms the input equation into

$$y^3 + 3py + 2q = 0$$

where

$$p = 3ac - b^2/9a^2, \quad q = (b/3a)^3 - bc/6a^2 + d/2a$$

The solution of this equation depends on the signs of the discriminant $D = q^2 + p^3$ and of parameter p. However, since p and q are fairly complex functions of ξ, numerical analysis on a computer was substituted for the cumbersome, if at all possible, analytical solution.

The results of calculations of $\alpha_* = \alpha_*(\xi)$ for various values of γ are shown in the Fig. 1, where curves 1, ..., 9 correspond to the following values of the adiabatic exponent:

	1	2	3	4	5	6	7	8	9
$\gamma=$	1.00001	1.05	1.1	1.2	1.3	1.4	1.666667	2	3

As expected, the [functional] dependence of $\alpha_*(\xi)$ for $\gamma = 1.4$ coincides numerically with that given in [10]. The calculations show the increasing effect of the adiabatic exponent on the critical incidence angle with increasing amplitude of the incident shock. Thus the effect of the adiabatic exponent is particularly strongly pronounced in strong incident waves in the case of one-dimensional (normal) as well as in that of "oblique" (inclined) reflections. (Note that $\alpha_* = 89.6°$ for $\gamma = 1.00001$ and $\xi = 0$.)

A further remarkable property should be noted. It appears that for $\xi = 0.763$, and independently of the adiabatic exponent, we virtually always have $\alpha_* = 50.8°$.

In the limit case of weak shock waves the problem can be analytically investigated. In fact, for $\xi \to 1$ Eq. (4.1) becomes

$$2(\gamma + 1)(1 - \xi)x^3 - \gamma x^2 - 2\gamma x - \gamma = 0, \quad (4.2)$$

Its approximate solution can be written in the form

$$x = [2(1 + 1/\gamma)(1 - \xi)]^{-1}.$$

Hence

$$\alpha_* = \mathrm{arc\,tg}\,[2(1 + 1/\gamma)(1 - \xi)]^{-1/2}. \quad (4.3)$$

Expansion of the right-hand side of (4.3) into series in $(1 - \xi)^{1/2}$ yields

$$\alpha_* = {}^1/_2\pi - \sqrt{2(1 + 1/\gamma)(1 - \xi)}, \quad (4.4)$$

correct to within the first term of the expansion. This shows, in particular, that α_* depends rather weakly on γ.

We further note that at the limit of strong shocks ($\xi \to 0$) the coefficients of Eq. (4.1) depend only on γ, and are reduced to

$$a = \gamma + 1, \quad b = \frac{2(\gamma+1)\gamma}{\gamma-1} - \frac{(\gamma-1)^2}{8\gamma},$$

$$c = \frac{(3\gamma+1)(\gamma+1)^2}{4\gamma(\gamma-1)}, \quad d = -\frac{(\gamma+1)^4}{8\gamma(\gamma-1)^2}.$$

The case of $\gamma = 1$ or $f = \infty$, when compression in a strong shock is infinitely great, is of particular interest. For γ close to 1 the equation for $x = \tan^2\alpha_*$ becomes

$$2x^3 + \frac{4}{\gamma-1}x^2 + \frac{4}{\gamma-1}x - \frac{2}{(\gamma-1)^2} = 0. \quad (4.5)$$

It follows from (4.5) that for $\gamma \to 1$ it can only be satisfied when $x = \infty$, i.e., when $\alpha_* = 1/2$ (for any finite x the left-hand side of the equation would tend to $-\infty$. Hence a regular reflection os such shoch is possible at any angle of incidence.

This can be ascertained by considering the shock polar curve, which for $M = \infty$ and $f = \infty$ degenerates into a circle passing through the coordinate origin, thus ensuring the deflection of flow by any angle.

LITERATURE CITED

1. R. C. Tolman, The Principles of Statistical Mechanics [in Russian], Clarendon Press, Oxford, 1938.
2. A. Sommerfeld, Lectures on Theoretical Physics, Vol. 2: Mechanics of Deformable Bodies [Russian translation], Izd-vo inostr. lit., Moscow, 1954.
3. K. P. Stanyukovich, Unstable Motions of Continuous Media [in Russian], Gostekhizdat, Moscow, 1955.

4. V. A. Belokon', V. A. Proskuryakov, and A. I. Petrukhin, "The penetration of a strong shock wave into a wedge-shaped cavity," ZhÉTF, 48, no. 1, 1965.
5. T. V. Bazhenova, L. G. Gvozdeva, Yu. S. Lobastov, I. M. Naboko, G. G. Nemkov, and O. A. Predvoditeleva, Shock Waves in Real Gases [in Russian], Nauka, Moscow, 1968.
6. V. A. Belokon', A.I. Petrukhin, and V. A. Proskuryakov, "On the penetration of a strong shock wave into a wedge-shaped cavity," Summaries of Reports to the 2nd All-Union Conference on Theoretical and Applied Mechanics [in Russian], Nauka, Moscow, 1964.
7. R. Courant and K. Friedrichs, Supersonic Flow and Shock Waves [Russian translation], Izd. inostr. lit., Moscow, 1950.
8. J. Neumann, "Oblique reflection of shocks," US Navy Dept., Ordnance Bureau, Expl. Res., Report 12, 1943.
9. J. Neumann, Collected Works, Vols. 1-6, Pergamon Press, Oxford, 1961-63.
10. L. D. Landau and E. M. Lifshitz, Mechanics of Continuous Media [in Russian], Gostekhteoretizdat, Moscow, 1954.